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Vibration of multilayer cantilever beams towards piezoelectric power harvesters

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ABSTRACT

This paper investigates the vibration characteristics of a commonly employed mechanical structure, the cantilever beam, concerning its potential for direct electricity harvesting from piezoelectric crystals. Piezoelectric materials are renowned for their ability to generate electric charges when subjected to mechanical stress. To ensure continuous current generation, these materials require sustained excitation by external forces, typically achieved through vibration. However, piezoelectric crystals lack sufficient elasticity, necessitating attachment to a structurally conducive and easily vibratable framework. The cantilever beam, renowned for its simplicity and widespread use, serves as an ideal platform for this purpose. Layers of piezoelectric material (PZT5H) are affixed to brass-based cantilever beams to create various multilayer configurations. External forces are then applied at the free end of the beam to induce vibration. Given that the harvested power from PZT5H crystals correlates with the mechanical stress they experience, achieving optimal deformation is paramount. This is accomplished by leveraging the resonance effect of vibration, wherein the vibration modes and natural frequencies of the multilayer PZT5H beams must be thoroughly characterized. To this end, numerical methods and Finite Element Analysis via Abaqus software are employed. The vibrations of the brass base layer, as well as single-sided and double-sided PZT5H beams, are analyzed across four distinct mode shapes and corresponding natural frequencies. The study culminates in a comprehensive examination of the relationship between natural frequencies and dimensional parameters of the beams. Ultimately, this research offers valuable insights into the vibration behavior of cantilever beams, laying the groundwork for the development of efficient power harvesting devices utilizing mechanical vibration sources or tailored to specific applications.

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1. Introduction

Piezoelectric materials have garnered significant attention in the realm of energy harvesting due to their unique ability to convert mechanical vibrations into electric power. This phenomenon is based on the piezoelectric effect, where certain materials, such as crystals or ceramics, generate an electric charge in response to mechanical stress, which was confirmed by Jacques Curie and Pierre Currie. The versatility of piezoelectricity is evident in its applications, ranging from sensors to actuators. However, one of its most promising applications lies in the generation of electric power. By incorporating piezoelectric materials into devices like sensors, wearable technology, or infrastructure components subject to vibration or movement, it becomes possible to harness ambient mechanical energy and convert it into electrical power. This capability holds immense potential for creating self-sustaining systems, reducing reliance on traditional power sources, and exploring innovative solutions for powering remote or low-energy devices. As research in material science and engineering advances, so too does the potential for optimizing piezoelectric technologies, opening new avenues for sustainable and decentralized energy harvesting (Dineva et al., 2014).

Piezoelectric materials offer a wide range of possibilities for power generation across various mechanical systems, making them adaptable to diverse applications. Membrane structures, vibrational energy harvesters (including springs and oscillating structures), and rotating mechanisms like gears and turbines can all capitalize on piezoelectricity to transform mechanical vibrations and motions into electrical power (Ericka et al., 2005; Janphuang et al., 2014; Karami et al., 2013; Wang et al., 2010; Wu et al., 2018). Besides, the integration of piezoelectric materials into textiles enhances the potential for wearable energy harvesting, utilizing body movements as a source of power (Ali et al., 2019; Delnavaz et al., 2014; Sun et al., 2019).

The cantilever beam, however, stands out as the preferred mechanism for harvesting energy from piezoelectric materials due to its inherent structural advantages that align with the piezoelectric effect (Repetto et al., 2012). Layers of piezoelectric materials are bonded to one or both sides of the beam, enhancing the energy conversion efficiency by maximizing the exposure of the piezoelectric materials to mechanical deformations. The asymmetric structure of the cantilever beam ensures that when subjected to external forces or vibrations, it undergoes controlled bending motions, including strain in the attached piezoelectric layers. By affixing the materials to one or both sides, the beam takes advantage of both tensile and compressive stresses during deformation, significantly increasing the overall strain experienced by the piezoelectric layers. Additionally, the simplicity of the cantilever beam design, combined with the enhanced strain distribution achieved through the layered configuration, makes it a promising mechanism for piezoelectric energy harvesting.

2. Methodology

2.1. Piezoelectric materials and piezoelectric effect

Piezoelectric materials generate an electric charge through the piezoelectric effect, which occurs in certain crystals and ceramics, such as quartz, lead zirconate titanate (PZT), and others, that have a non-centrosymmetric crystal structure. Positive and negative charges are not perfectly aligned, creating a dipole moment within the crystal. When mechanical stress or deformation is applied to the piezoelectric material, it causes a shift in the positions of the positive and negative charges within the crystal lattice. The movement of charges creates an electric potential across the material, resulting in the generation of an electric charge (Dineva et al., 2014). This charge can be harnessed as an electrical current when the piezoelectric material is part of a closed circuit.

Figure 1 illustrates the fundamental principle of charge generation through the application of mechanical forces on the piezoelectric material. The 33 mode, characterized by compressive forces, results in a voltage direction parallel to the applied force. In contrast, the 31 mode, a transverse mode, yields a harvested voltage perpendicular to the force direction. A comparative analysis of the power generation efficiency between these two modes reveals that the 31 mode is notably more advantageous for harvesting applications (Roundy et al., 2003). Consequently, the cantilever beam structure emerges as an ideal configuration for inducing the 31 mode during vibration, thereby enhancing the efficiency of power harvesting applications.

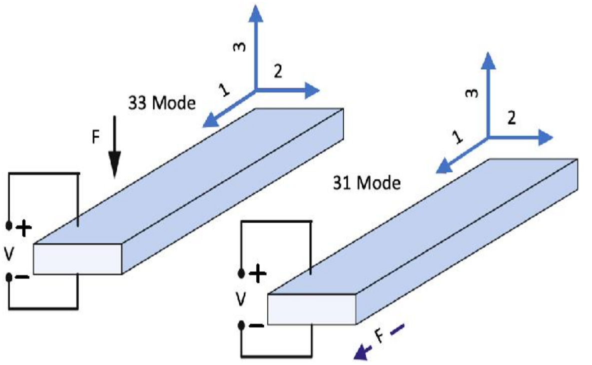

Figure 1. Mechanical strain modes of piezoelectric material in charge generation (Sezer & Koç, 2020).

2.2. Cantilever beams vibration and frequencies

In this paper, the cantilever beam is described as a long, wide, and relatively thin structure with one end fixed while the other end free, as sketched in Figure 2a.

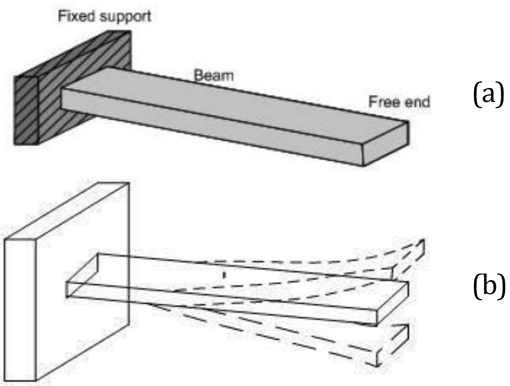

Figure 2. Single layer thin cantilever beam (Gurusharan et al., 2021). (a) A typical cantilever beam; (b) Cantilever beam deflection under end load.

When a point load P is applied to the free end of the beam, the beam undergoes deflection, forming a curved profile, Figure 2b. The displacement of the free end is calculated in Equation (1):

$$y(x) = -\frac{PL^3}{3EI} \qquad (1)$$

Where: L - length of the beam, m; E - elastic modulus of the material, N/m²; I - moment inertia of the beam, m⁴.

Since the cross-section of the thin beam is rectangular, the moment of inertia of area is defined as in Equation (2):

$$I = \frac{wh^3}{12} \qquad (2)$$

In which: w is the width and h is the thickness of the beam in meters, respectively.

When the force is no longer applied to the beam, it will revert to its initial configuration. Nevertheless, due to the moment of inertia, the beam will undergo vibrations around its equilibrium position. The corresponding Equation is given in Equation (3):

$$EI\frac{\partial^4 y}{\partial x^4} = -\mu\frac{\partial^2 y}{\partial t^2} \qquad (3)$$

With $\mu = \rho A$ is the linear mass density of the beam, in which ρ is the mass density, kg/m³, and A is the cross-sectional area of the beam, m².

That means the displacement depends on both position and time.

If the cantilever beam has a uniform cross-section, the formula for the natural frequencies can be expressed by Equation (4):

$$\omega_n = \frac{(2n-1)\pi}{L^2}\sqrt{\frac{EI}{\rho A}} \quad n = 1, 2, 3, \ldots \qquad (4)$$

Where: n - the mode shape of the vibration.

Then the general solution for the vibration is given by Equation (5):

$$y(x) = A_1\,cosh(\beta_n x) + A_2\,sinh(\beta_n x) + A_3\,cos(\beta_n x) + A_4\,sin(\beta_n x) \qquad (5)$$

Where: A_1, A_2, A_3, A_4 - constants defined from the specified boundary conditions; $\beta_n = \sqrt[4]{\frac{\mu\omega_n^2}{EI}}$, with ω_n - the natural frequencies of the beam.

Each solution for displacement is termed a mode, and the configuration of the displacement curve is referred to as a mode shape, as depicted in Figure 3 below.

If another material layer is attached to the beam, then the mass density, elastic modulus and stiffness of the beam will change, which leads to the change in the natural frequency.

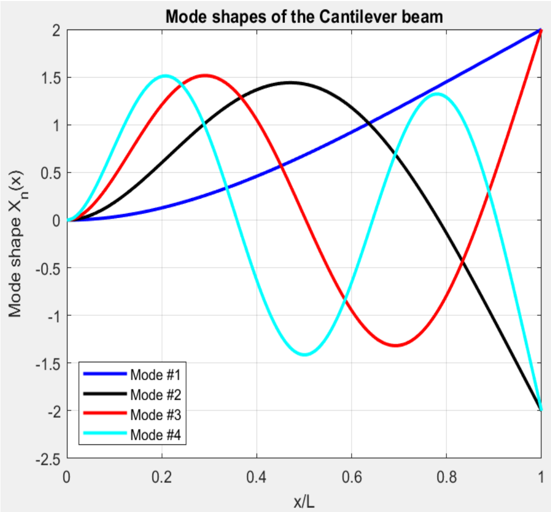

Figure 3. Typical mode shapes of a vibrating cantilever beam.

The determination of the equivalent stiffness of a multi-layer beam is imperative for assessing its overall rigidity, considering the collective influence of each constituent layer. This calculation typically relies on the distinctive properties of individual layers. The equivalent stiffness (k_{eq}) for a beam comprised of n parallel layers is indeed the sum of the stiffness of each layer (k_i). Mathematically, this is expressed by Equation (6):

$$k_{eq} = \sum_{i=1}^{n} k_i \tag{6}$$

The stiffness (k_i) of a homogeneous cantilever beam can be calculated using Equation (7):

$$k_i = \frac{E_i I_i}{L_i^3} \tag{7}$$

The equivalent elastic modulus, however, is computed from the individual component elastic modulus (E_i) and the corresponding cross-sectional area (A_i) by Equation (8) below:

$$E_{eq} = \frac{\sum_{i=1}^{n} E_i A_i}{\sum_{i=1}^{n} A_i} \tag{8}$$

Later, the equivalent mass density (ρ_{eq}) is derived as the summation of the mass density associated with each layer within the beam (ρ_i), as expressed in Equation (9):

$$\rho_{eq} = \sum_{i=1}^{n} \rho_i \tag{9}$$

In the pursuit of power generation using piezoelectric materials, an imperative consideration lies in the correlation between the deflection magnitude and generated power. Specifically, the utilization of resonance becomes paramount. The optimal harvesting of power from a vibrational piezoelectric beam occurs when its natural frequency aligns with the frequency of the input mechanical power source, leading to maximal deflection. This strategic alignment enhances the efficiency of power generation, as the piezoelectric material attains peak strain levels, resulting in elevated output voltage and power. Consequently, meticulous tuning of the natural frequency of the piezoelectric beam to synchronize with the frequency of the mechanical vibrations is integral for the optimization of power harvesting systems.

3. Vibration of multi-layer piezoelectric cantilever beams

The piezoelectric power-harvesting beams are constructed by affixing layers of piezoelectric material onto an elastic beam. The design flexibility allows for the customization of power generation based on specific requirements. For instance, to achieve the desired power output, one layer of piezoelectric material can be adhered to a single side of the beam, forming a two-layer cantilever beam. Alternatively, for increased power generation, two pieces of piezoelectric material can be applied to both sides of the beam, resulting in the creation of a three-layer sandwich cantilever beam. This modular approach provides a versatile framework for tailoring the power harvesting system to meet varying performance objectives.

The beams investigated in this paper, both the single and double-sided configurations, are fabricated by adhering layers of PZT5H (Lead Zirconate Titanate Piezoelectric crystal) onto elastic elements crafted from brass. Detailed technical information about the materials is presented in Table 1, while the structural composition of the beam is visually elucidated in Figure 4.

Table 1. Material technical data: PZT5H plate and Brass bar (Typical values).

No	Parameters	Symbol	Units	Values
PZT5H (Lead Zirconate Titanate)				
1	Relative Dielectric Constant (at 1 kHz)	K		3200
2	Piezoelectric charge (Displacement coefficient)	d_{31}	m/V	-320×10^{-12}
3		d_{33}	m/V	650×10^{-12}
4	Coupling coefficient	k_{31}	μCoul/cm²	0,43
5		k_{33}	μCoul/cm²	0,75
6	Density	ρ	kg/m³	7800
7	Thermal expansion	α	ppm/°C	3,5
8	Poisson's Ratio	u		0,31
9	Elastic modulus	E	N/m²	5×10^{10}
Brass				
10	Density	ρ	kg/m³	8500
11	Thermal expansion	α	ppm/°C	3,5
12	Poisson's Ratio	u		0,32
13	Elastic modulus	E	N/m²	10×10^{10}
14	Maximum operating temperature	T	°C	750

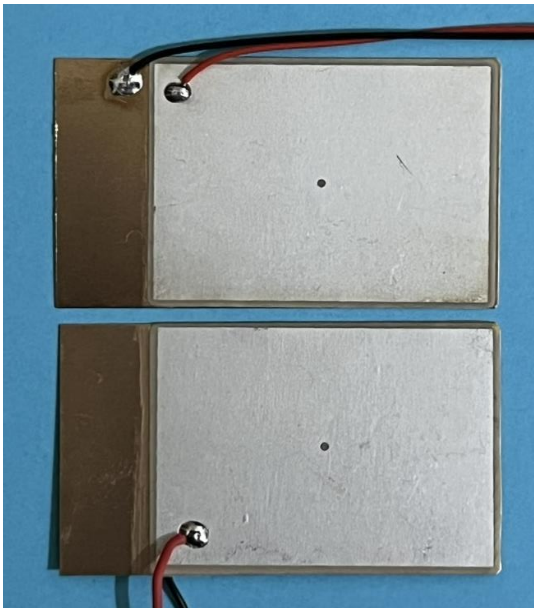

Figure 4. Multi-layer piezoelectric beams (upper: one-sided PZT beam; lower: double-sided PZT beam).

The PZT5H plate dimensions are 40 × 30 × 0,2 mm, and the brass beam dimensions are 50 × 30 × 0,2 mm, respectively. Thin layers of epoxy, approximately 0,02 mm in thickness, have been applied between the two materials for adhesive purposes, however, for modelling, these epoxy layers can be disregarded.

Utilizing the provided Equation (2), the moment of inertia (I) for both the PZT5H layer (I_P) and brass beam (I_B) can be calculated based on their respective cross-sectional dimensions. Given that they share the same width and thickness, the relationship is calculated as in Equation (10):

$$I_P = I_B = \frac{wh^3}{12} = \frac{0{,}03 \times 0{,}0002^3}{12} = 2.10^{-14}, m^4 \quad (10)$$

The PZT5H plate is affixed at one end of the beam, and subsequently, the entire beam is clamped at the other end of the plate, forming a cantilever beam, as shown in Figure 5. This configuration ensures that both the plate and the beam share an equivalent length (L) for stiffness calculation, with $L_P = L_B$ = 40 mm = 0,04 m.

As a result, the stiffness of the PZT5H plate (k_P) in Equation (11) and the beam (k_B) in Equation (12) will be computed from Equation (7):

$$k_P = \frac{E_P I_P}{L_P^3} = \frac{5.10^{10} \times 2.10^{-14}}{4.10^{-2}} = 15{,}625\,, Nm \quad (11)$$

$$k_B = \frac{E_B I_B}{L_B^3} = \frac{10.10^{10} \times 2.10^{-14}}{(4.10^{-2})^3} = 31{,}25\,, Nm \quad (12)$$

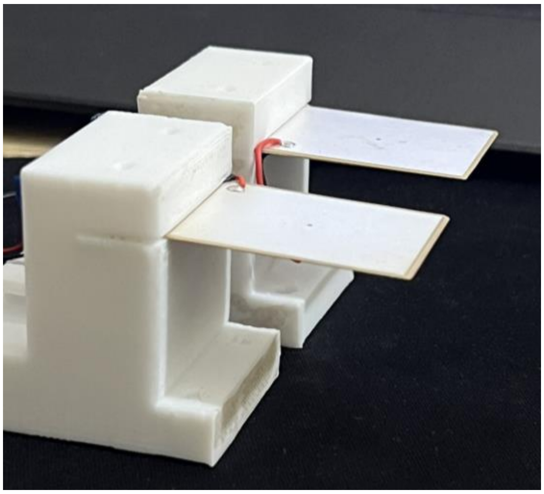

Figure 5. One-sided (left) and double-sided (right) PZT5H cantilevered beams.

Subsequently, the equivalent stiffness of both the one-sided (k_s) in Equation (13) and double-sided (k_d) in Equation (14) cantilever beams is determined utilizing Equation (6):

$$k_s = \sum_{i=1}^{n} k_i = k_P + k_B = 46{,}875\ , Nm \quad (13)$$

$$k_d = \sum_{i=1}^{n} k_i = k_P + k_B + k_P = 62{,}5\ , Nm \quad (14)$$

Given that each layer in the beam processes an identical cross-sectional area (A_i) characterized by the same width and thickness, the expression for the equivalent Young's modulus, as defined in Equation (8), can be simplified as in Equation (15) bellows:

$$E_{eq} = \frac{\sum_{i=1}^{n} E_i}{n} \quad (15)$$

Then the equivalent Young's modulus of one-sided (E_s) in Equation (16) and double-sided (E_d) in Equation (17) beams are determined:

$$E_s = \frac{E_P + E_B}{2} = \frac{5.10^{10} + 10.10^{10}}{2} = 7{,}5 \times 10^{10}\ , N/m^2 \quad (16)$$

$$E_d = \frac{E_P + E_B + E_P}{3} = \frac{2 \times 5.10^{10} + 10.10^{10}}{3} = 6{,}667 \times 10^{10}\ , N/m^2 \quad (17)$$

The equivalent mass densities are then computed using Equation (9), shown in Equation (18) and Equation (19) respectively:

$$\rho_s = \rho_P + \rho_B = 7800 + 8500 = 16300\ , kg/m^3 \quad (18)$$

$$\rho_d = 2\rho_P + \rho_B = 2.7800 + 8500 = 24100\ , kg/m^3 \quad (19)$$

The numerical results, depicted in Figure 6, illustrate the first four mode shapes of vibration for the base brass bar cantilever beam, the one-sided

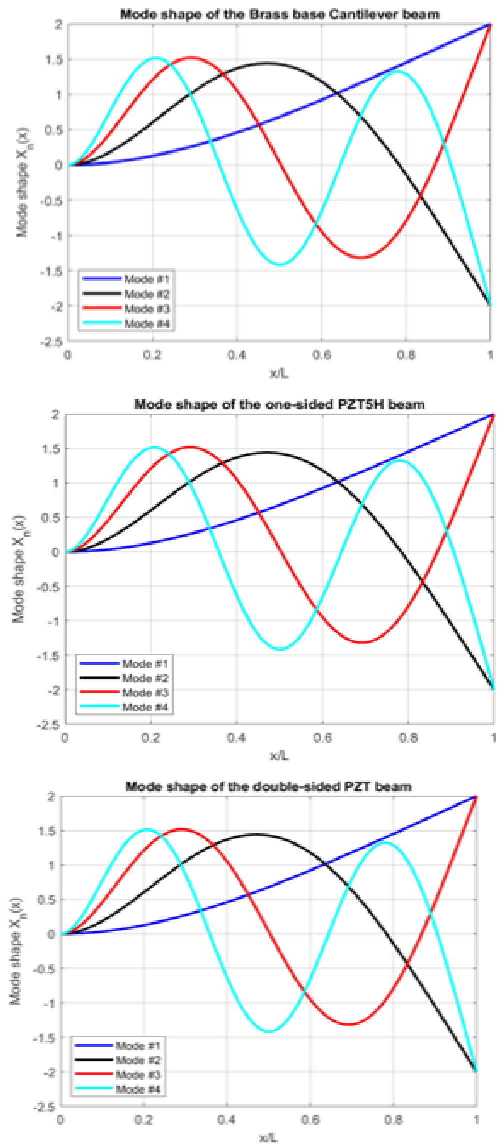

Figure 6. First four mode shapes of the base brass bar cantilever beam, one-sided and double-sided PZT5H beams, respectively.

PZT5H beam, and the double-sided PZT5H beam, illustrate distinct bending patterns. Each mode introduces additional complexities, with the second, third, and fourth modes featuring more modal points, reflecting the evolving and intricate nature of the beam's vibrational behavior. It is evident that all three beams exhibit an identical mode shape of free vibration, indicating a uniform deformation pattern. This uniformly implies consistency in structural geometry and material properties across the three beams, with the primary influence on mode shape attributed to these factors. Remarkably, the vibration mode of the beam is predominantly determined by the elastic base layer of brass, while the contribution of the PZT5H layer is found to be minimal. This observation underscores the significance of structural geometry and base material in influencing the vibrational characteristics of the beams. However, it is noteworthy that the natural frequencies reveal variations, as evident in Table 2. This discrepancy implies the impact of differing stiffness and mass distribution on the vibrational responses of the beams. The data in Table 2 provides a clear insight into how these structural parameters contribute to the observed differences in natural frequencies among the studied beams.

The Finite Element Method (FEM) is a crucial tool for analyzing beam structures. By breaking down complex structures into manageable elements, FEM allows for a detailed examination of the dynamics of beams under various conditions. This method provides insights into critical factors such as stress distribution, deformations, and natural frequencies. FEM predicts beam response to different conditions, streamlining analysis for design optimization and ensuring a comprehensive understanding of structural integrity.

Abaqus stands out as a premier software in FEM, enabling the precise simulation of complex structural mechanics. Its user-friendly interface and robust features make it a go to choice for engineers, facilitating detailed modeling, material property input, and accurate simulation of diverse loading scenarios. The simulated results from Abaqus will be compared to the numerical results above to validate the beams' behavior.

Figure 7 presents the Abaqus simulated FEM results of the vibration of the PZT cantilever beam, showcasing unique deformation patterns. In the first mode shape, the cantilever beam exhibits a simple bending pattern. For the second mode, an additional nodal point emerges in the beam's structure, introducing a more complex bending behavior. The third and fourth modes further elaborate on this complexity, with multiple nodal points contributing to the evolving vibrational dynamics of the beam. The simulation results align entirely with the numerical calculations in terms of the mode shape of vibration. Notably, the dominant contribution to the vibration pattern comes from the elastic brass beam, overshadowing the influence of the PTZ layers. Furthermore, the equivalent natural frequencies for each mode shape closely match the calculated results, as shown in Table 2.

In short, the close correspondence between the numerical results and simulated results attests to the reliability of the analytical methods. The agreement in mode shapes of vibration and equivalent natural frequencies, coupled with the dominance of the elastic brass beam in shaping vibrations, reinforces the validity of the finding. This convergence enhances confidence in the accuracy of our approach, laying a strong foundation for its application in structural analysis and design optimization.

Table 2. Natural frequencies of PTZ beam vibration.

Mode shape	Elastic Brass Beam		One-sided PZT5H beam		Double-sided PZT5H beam	
	Numerical frequency	Simulated frequency	Numerical frequency	Simulated frequency	Numerical frequency	Simulated frequency
1	69,26	69,26	86,63	86,61	100,76	100,74
2	434,04	433,98	542,89	542,59	631,42	630,64
3	1215,33	1214,9	1520,1	1518,1	1768,00	1762,8
4	2381,57	2380,1	2978,79	2971,5	3464,58	3445,7

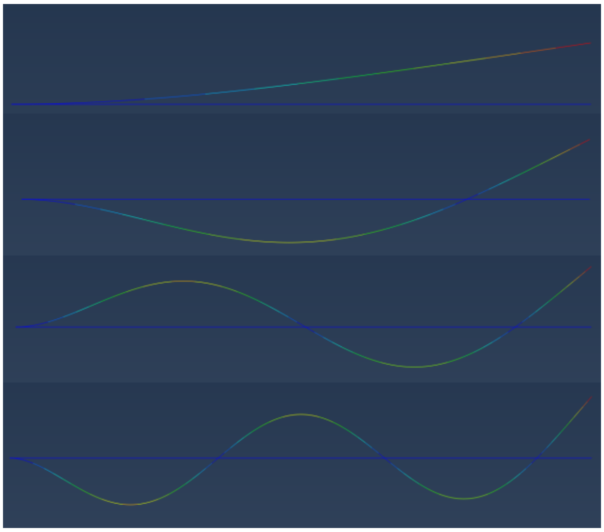

Figure 7. The first four vibration mode shapes of the PZT5H beams obtained through Abaqus simulation.

Figure 8 presents a comprehensive analysis of the correlation between the natural frequencies across diverse mode shapes and the dimensional ratios of beam thickness to the square of beam length, as per Equation 4. Solid lines delineate this relationship for one-sided PZT beams, while dashed lines denote the equivalent for double-sided PZT beams. Elevated natural frequencies correspond to higher ratio and/or mode, and one-sided PZT beams exhibit slightly faster vibrations than their double-sided counterparts. The primary objective of this research is to discern and articulate these natural frequencies precisely. By aligning these frequencies with the power source,

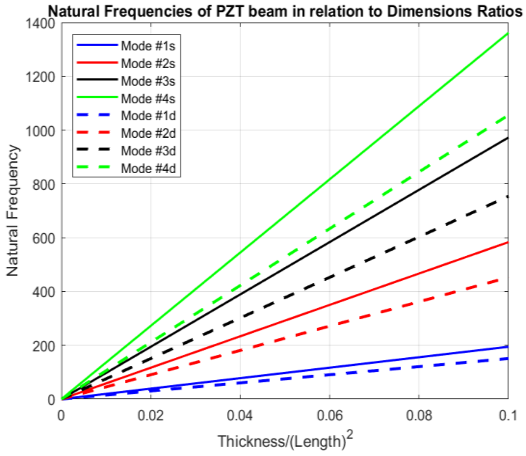

Figure 8. Exploring the Correlation: Natural Frequencies in Relation to Dimensional Ratios of PZT Beams.

we aim to strategically determine the optimal beam type and dimensions, leveraging resonance effects to enhance the deformation of the PZT layers and maximize energy output from the device.

4. Conclusion

This study investigates the power-harvesting potential of piezoelectric cantilever beams, emphasizing their unique ability to convert mechanical vibration into electric power. The cantilever beam structure, particularly with PZT5H layers on brass, proves to be an efficient mechanism for energy harvesting. The research delves into the piezoelectric effect, vibration characteristics, and the role of resonance in optimizing power generation.

Through numerical analysis and Finite Element Method (FEM) simulations using Abaqus software, the study validates the dominance of the base material in determining the vibration mode shapes. The findings emphasize the importance of strategic tuning of natural frequencies for optimal power generation. This insight contributes to the design and optimization of piezoelectric cantilever beams, offering potential applications in sustainable and decentralized energy harvesting.

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Contributions of authors

Luan Cong Doan - methodology, writing, review & editing; Giap Van Doan - writing, review & editing, supervision.

References

Ali, F., Raza, W., Li, X., Gul, H., & Kim, K.-H. (2019). Piezoelectric energy harvesters for biomedical applications. *Nano Energy*, *57*, 879-902. https://doi.org/10.1016/j.nanoen.2019.01.012.

Delnavaz, A., Voix, J. J. (2014). Flexible piezoelectric energy harvesting from jaw movements. *Smart Materials and Structures, 23*(10), 105020, IOP Science.

Dineva, P., Gross, D., Müller, R., Rangelov, T., Dineva, P., Gross, D., Rangelov, T. (2014). *Piezoelectric materials*. Springer.

Ericka, M., Vasic, D., Costa, F., Poulin, G., & Tliba, S. (2005). Energy harvesting from vibration using a piezoelectric membrane. *Journal de Physique IV* (Proceedings).

Gurusharan, Shegunashi, G., Adarsh, P., & Gurunagendra, G. R. (2021). Influence of envelope and core angles on natural frequency of a cantilever beam - A finite element approach. *Materials Today: Proceedings*, *47*, 2237-2240. https://doi.org/10.1016/j.matpr.2021.04.192.

Janphuang, P., Lockhart, R. A., Isarakorn, D., Henein, S., Briand, D., & de Rooij, (2014). Harvesting energy from a rotating gear using an AFM-like MEMS piezoelectric frequency up-converting energy harvester, *Journal of Microelectromechanical System, 24*(3), 742-754. IEEE.

Karami, M. A., Farmer, J. R., & Inman, D. J. (2013). Parametrically excited nonlinear piezoelectric compact wind turbine, *Journal of Renewable Energy, 50*, 977-987. Elsevier.

Repetto, C. E., Roatta, A., & Welti, R. J. (2012). Forced vibrations of a cantilever beam. *European Journal of Physics*, *33*(5), 1187. https://doi.org/10.1088/0143-0807/33/5/1187.

Roundy, S., Wright, P. K., & Rabaey, J. (2003). A study of low level vibrations as a power source for wireless sensor nodes, *Journal of Computer Communication, 26*(11), 1131-1144. Elsevier.

Sezer, N., & Koç, M. (2020). A Comprehensive Review on the State-of-the-Art of Piezoelectric Energy Harvesting. *Nano Energy*, *80*, 1-25. https://doi.org/10.1016/j.nanoen.2020.105567.

Sun, C., Shang, G., Wang, H. (2019). On piezoelectric energy harvesting from human motion, *Journal of Power and Energy Engineering* *7*(1), 155-164.

Wang, D. A., & Ko, H. H. (2010). Piezoelectric energy harvesting from flow-induced vibration. *Journal of Micromechanics and Microengineering*, 20(2), 025019.

Wu, Y., Qiu, J., Zhou, S., Ji, H., Chen, Y., & Li, S. (2018). A piezoelectric spring pendulum oscillator used for multi-directional and ultra low frequency vibration energy harvesting, *Journal of Applied Energy*. *231*, 600-614. Elsevier.